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π^--*D* Scattering at 142 MeV*†

E. G. PEWITT, T. H. FIELDS,‡ G. B. YODH,§ J. G. FETKOVICH, AND M. DERRICK‖
Department of Physics, Carnegie Institute of Technology, Pittsburgh, Pennsylvania
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The interactions of 142-MeV π^- mesons with deuterium has been studied using a 15-cm bubble chamber in a 12.7-kG field. The angular distributions for elastic and inelastic scattering were measured. The elastic differential cross section for scattering angles greater than 90° is not in agreement with theoretical calculations based on the impulse approximation. The total cross section for neutral products ($2n$, $2n+\gamma$, $2n+\pi^0$) was measured to be 37.2 ± 2.8 mb. The sum of the separate cross sections gave the π^-+d total cross section to be 183 ± 7 mb.

INTRODUCTION

A THREE-body problem of special importance to elementary-particles physics is that of a high-energy particle interacting with two nucleons bound as a deuteron. A satisfactory solution of the many theoretical complications of this problem will resolve the uncertainties involved in the use of deuterium as a free-neutron target, and will allow experimental measurement of finer details of the deuteron wave function and of two-body scattering amplitudes for "unphysical" momentum transfers. This experiment, a measurement of pion scattering from deuterium, provides a particularly severe test of theoretical models, since the pion energy is close to that of the (3, 3) resonance.

As the low-energy pion-nucleon interaction is fairly well understood both experimentally and theoretically, one tries to apply impulse-approximation methods to the pion-deuteron problem, treating the three-body π-d interaction in terms of two two-body pion-nucleon interactions. Several theoretical investigations of π-d scattering using the impulse approximation (IA hereafter) have been carried out.[1–8] Experimental information,[9–13] on π-d interactions has been compared with these calculations, and depending somewhat upon the particular calculation, the agreement of theory with experiment was found to be satisfactory. However, one cannot regard this agreement as more than a coarse test of the theory because (a) the accuracy of the experiments was poor in the regions where the shortcomings of the theoretical models would be expected to show up, and (b) several theoretical calculations gave results which differed from one another by amounts great enough to raise general questions as to their validity.

Recently, Pendleton[14] has carried out the most complete IA calculation of π-d elastic scattering to date. This calculation takes into account the tensor force and the repulsive-core terms of the nucleon-nucleon potential and also double-scattering effects. We undertook an experimental study of π-d interactions in a bubble chamber to provide data of sufficient accuracy to allow reasonably detailed comparison with the theoretical predictions.

The reactions considered in the data analysis were

$$\pi^-+d\to\pi^-+d\,,\qquad \text{elastic}\,,\qquad (1)$$

$$\to\pi^-+p+n\,,\qquad \text{inelastic}\,,\qquad (2)$$

$$\to n+n+\pi^0\,,\qquad \text{charge exchange}\,,\qquad (3)$$

$$\to n+n\,,\qquad \text{absorption}\,,\qquad (4)$$

$$\to n+n+\gamma\,,\qquad \text{radiative absorption.}\qquad (5)$$

Elastic scattering was of the most direct interest, for reasons given above, and its two-body final state provided a convenient signature. The only significant background arose from inelastic events. The use of negative rather than positive poins minimized the number of inelastic events with visible recoil tracks, since the resonant cross section for (π^-p, π^-p) is only one-ninth that of (π^+p, π^+p). The energy of 142 MeV was chosen as a compromise between scattering with high-momentum transfers to be able to probe the deuteron structure at short-internucleon separations and having the recoil deuteron come to rest in the chamber volume for reasons of experimental convenience.

The accuracy aimed for was to determine the elastic

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† This paper is based upon a thesis presented by E. G. Pewitt to the Carnegie Institute of Technology in partial fulfillment of the requirements for the Ph.D. degree.
‡ Present address: Department of Physics, Northwestern University, Evanston, Illinois, and Argonne National Laboratory, Argonne, Illinois.
§ Present address: Department of Physics, University of Maryland, College Park, Maryland.
‖ Present address: Clarendon Laboratory, Oxford University, Oxford, England.
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[14] Hugh N. Pendleton, Phys. Rev., following paper, **131**, 1833 (1963).

FIG. 1. A typical exposure.

differential π-d scattering cross section at angles greater than 90° to about 10%. This would provide sufficiently accurate data to observe D-wave and repulsive-core effects (approximately 20%), assuming that the theory adequately accounted for other effects such as multiple scattering.

APPARATUS

Chamber

The bubble chamber,[15] which was piston expanded, operated at 32.1°K in a 12.7-kG field and had a repetition rate of one picture every three seconds. Two 35-mm stereo pictures were taken of each expansion. Each picture contained on the average twelve incident pions. A total of 80 000 usable pictures were taken of which 30 000 were analyzed. A typical picture is shown in Fig. 1.

The deuterium used in the chamber was obtained from Oak Ridge National Laboratory as heavy water which was electrolyzed by the National Bureau of Standards at Boulder, Colorado. The tritium to deuterium atomic ratio was approximately 10^{-14}. This estimate was made from measurements at ORNL, and it was consistent with the number of observed background bubbles resulting from the tritium β decays. When the chamber was half exhausted at the end of the run, a gas sample was taken. Mass spectrometric analysis[16] of this sample showed 98.90 mole % of deuterium and 1.10 mole % of hydrogen. This hydrogen contamination resulted in a negligible correction in the measured cross sections.

[15] E. G. Pewitt, USAEC, New York Office Report NYO-9283, 1961 (unpublished); this report contains a description of the chamber as well as a more complete writeup of this experiment.

[16] Analysis made by Consolidated Electrodynamics Corporation, 300 North Sierra Madre Villa, Pasadena, California.

Beam

The mean energy and energy width of the pion beam traversing the deuterium was measured in three ways: (1) differential range curve of the beam before the bubble chamber was set in place; (2) curvature measurements of incident tracks in the chamber; (3) range and kinematic information of the elastic events (e. g., Fig. 5). These three methods were consistent with a mean beam energy of 142 MeV in the middle of the scanned region and an energy width of ± 8 MeV. The contamination of the beam was measured with a Čerenkov counter by Kovacik.[17] He found that it contained $6\pm1.5\%$ electrons and $5\pm1.5\%$ muons and, thus, $11\pm2\%$ total contamination.

DATA ANALYSIS AND RESULTS

Scanning and Measuring

Since the primary purpose of this experiment was to obtain accurate backward-elastic-scattering data, only the upstream half of the chamber was scanned for interactions. Thus, the downstream half was reserved to provide observable volume in which the recoil deuterons could stop. Thirty rolls (30 000 frames) were scanned for pion scatterings with no recoil (one-pronged events or 1 pr) and pion scatterings with a visible recoil track (two-pronged events or 2 pr). A 2-pr event might also arise from internal conversion of a γ ray from reactions (3) and (5) listed above. These electron pairs were in most cases easily distinguishable from reactions (1) and (2) by observing the curvature and ionization of the positive tracks. Two rolls were scanned for ends (0-pr events). These events resulted from reactions 3, 4, and 5.

Twenty-one of the thirty rolls were scanned twice. This gave a single scan efficiency of $93.5\pm1.0\%$ for all types of events.

Accepted events were restricted to those whose incident pion entered the chamber through a thin window, and the total number of incident pions was determined from a track count which included this same restriction.

The pictures were measured on an accumulating digitizer measuring machine.[18] Measurements on the film (demagnification of 6) corresponded to locating the center of the 150-μ-wide tracks in the chamber to within 45 μ. The procedure for measuring each view of a 2-pr event was to measure three fiducial marks, which may be seen in Fig. 1; the vertex of the event; and two other points on each of the three tracks in a prescribed order. One fiducial mark was remeasured to check against film movement and malfunctions in the electronics. The measuring of 1-pr and 0-pr events was very similar.

Elastic Events

The selection of elastic events from the combination of elastic and inelastic was accomplished by examining

[17] W. Kovaik, thesis, Carnegie Institute of Technology, 1960 (unpublished).

[18] T. Fields and R. Findley, Rev. Sci. Instr. **31**, 1312 (1960).

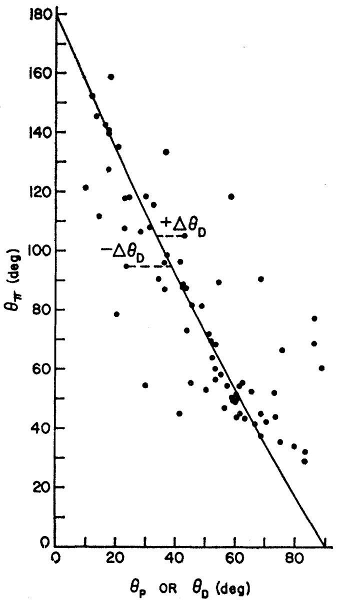

FIG. 2. Angular correlation of a sampling of 2-pr events. The line shown is for π-d elastic scattering at 140 MeV.

the angular correlation, coplanarity of the event, range of the recoil track, and curvature of the recoil track.

The angular correlation between pion-laboratory-scattering angle and the recoil-laboratory-scattering angle for one roll is given in Fig. 2. Also plotted is the two-body angular-correlation line for 140-MeV π mesons elastically scattered from deuterium. The distance a point was displaced in the θ_{recoil} direction was designated $\Delta\theta_D$, the sign of which is defined in Fig. 2. Figure 3 gives plots of $\Delta\theta_D$ for events with θ_π^{lab} between 30° and 180°. The curves are broadened by measuring error and by the spread in the incident beam energy.

A few events which satisfied the π-d angular correlation condition were observed to be noncoplanar outside of the limits allowed by measuring errors; the coplanarity error distribution was checked by comparison with that found by Deahl *et al.*[19] for $\pi^- + p$ elastic events.

For those events which satisfied the above criteria, a further selection was made, based upon recoil range, in the following manner. The apparent incident-pion energy E_π^{inc} was calculated for each event, given the range of recoil and the pion-lab-scattering angle, and assuming it to have been an elastic π-d scattering. The correlation among θ_π, range, and incident energy for elastic events is shown in Fig. 4. A distribution in E_π^{inc} was plotted for each angular range. These are Figs. 5 and 6. In addition to the spread in energy of the incident pions, the E_π^{inc} peaks were broadened by range straggling, measuring error in range of recoil, measuring error in the angle θ_π, and inclusion of some inelastic events.

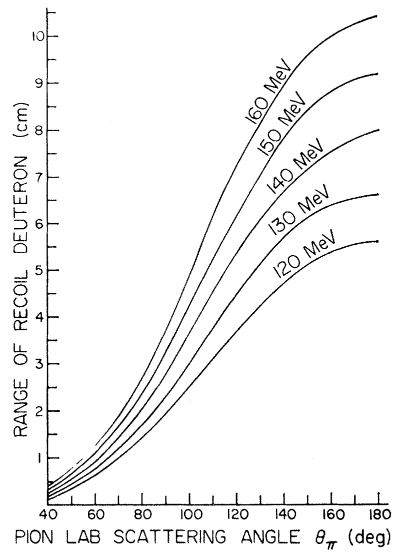

FIG. 4. Range of recoil deuteron in the bubble chamber ($\rho = 0.131$ gm/cc) as a function of pion-laboratory-scattering angle for several incident-pion energies.

The maximum error introduced into E_π^{inc} by range straggling was 0.3 MeV which is negligible. The error in measuring the length of a recoil track was found by remeasurement to be 0.15 mm, which corresponded to a negligible broadening of the E_π^{inc} peak for pion scattering angles greater than 40°. The measuring error in the pion scattering angle was of the order of 1°. This measuring error resulted in an error in E_π^{inc} of from 9 MeV in the 30° to 40° angular range to 0.6 MeV in the angular range 150° to 180°. The spread in the E_π^{inc} peak due to measuring errors in range and θ_π was reduced by remeasuring three-fourths of the events and plotting average values. The Gaussian fitted to the events in Fig. 5 is centered about 142 MeV with a width (standard deviation) of ±8.4 MeV. This beam-energy spread was equal within errors to that obtained from curvature measurements and also to that obtained from differential range data.

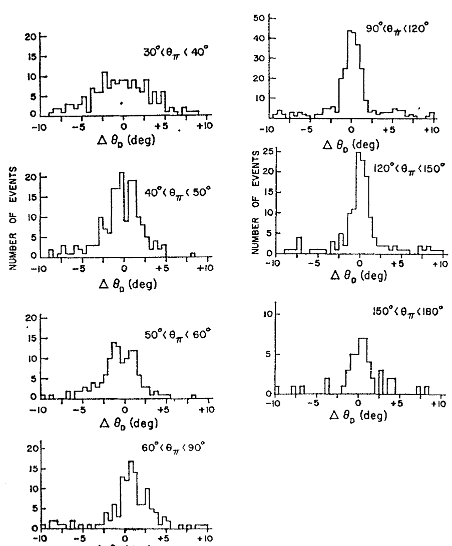

FIG. 3. Angular correlation $\Delta\theta_D$ for events with pion-scattering angle between 30° and 180°.

[19] J. Deahl, M. Derrick, J. Fetkovich, T. Fields, and G. B. Yodh, Phys. Rev. **124**, 1987 (1961).

Measurements were made of the average curvature of stopping recoils to determine if this means might be used to discriminate against the inelastic background of stopping protons. The results are shown in Fig. 7. The curves shown on Fig. 7 were calculated as follows: A program was written which calculated the film coordinates of stopping particles in liquid deuterium due to a 12.7-kG field. The initial energy was varied so that the resulting range was from 1 to 10 cm. Multiple scattering was neglected. The computed film coordinates of these stopping tracks were fed into the same geometry program used to calculate momentum of stopping tracks, and the results were corrected for multiple scattering according to the method described by Kim.[20] Corrections due to measuring errors were neglected since each stopping track was measured four times. The corrected momentum as a function of range for stopping proton and deuteron is plotted in Fig. 7.

Unambiguous inelastic events ($\Delta\theta_D > |10°|$) were used as a source of stopping protons. Their momentum versus range is plotted in 1-cm intervals in Fig. 7. An average of $1/p$ was found for the measurements in each

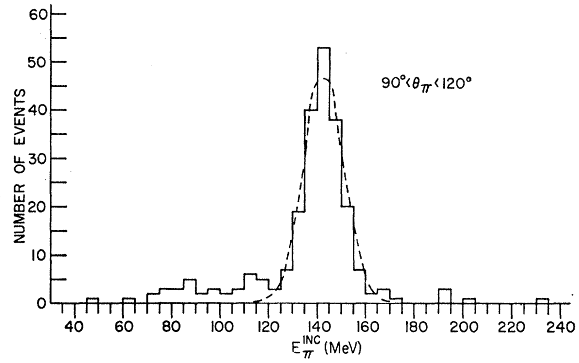

FIG. 5. Plot of apparent incident energy assuming the event to have been elastic for the angular range $90° < \theta_\pi < 120°$. The dashed curve is a Gaussian fit to the peaked histogram. All events on the $E_\pi^{\rm inc}$ plot were coplanar and had $\Delta\theta_D < |5°|$.

bin along with the standard deviation in $\langle 1/p \rangle_{\rm av}$. The deuteron points represent a like average of events which lay between 120 and 160 MeV on the $E_\pi^{\rm inc}$ plots. The proton points and the "elastic" points are clearly separate and show that the main body of recoils from

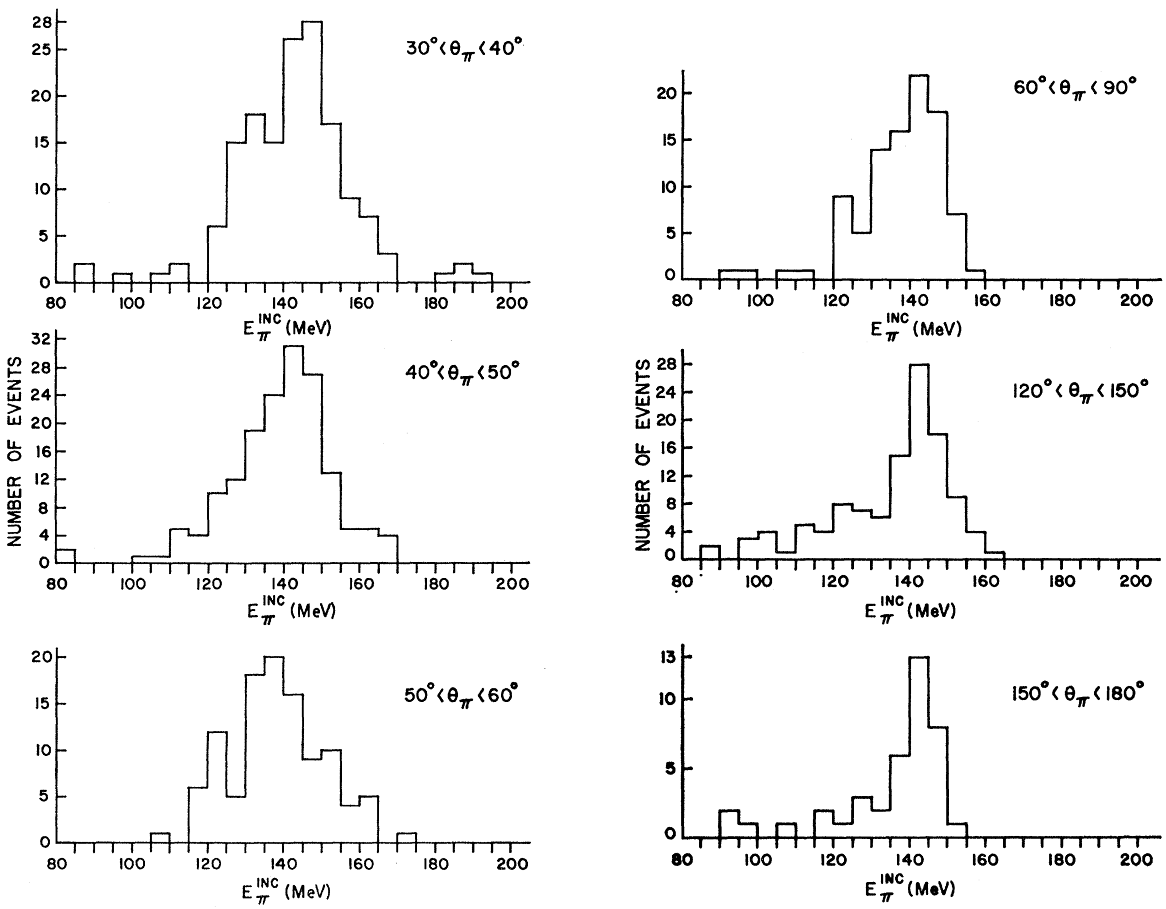

FIG. 6. Apparent incident energy for the remaining angular ranges.

[20] Y. B. Kim, Rev. Sci. Instr. **29**, 680 (1958).

Table I. Summary of elastic events.

Angular range of pion-lab angle (deg)	Number of Events: $E_\pi^{\rm inc}$ plot (recoil stops inside chamber)	Recoil scatters	Recoil stops outside chamber	Efficiency correction	Total	No. of tracks in the thin window on B view	Differential lab cross section $d\sigma/d\Omega$ (mb/sr)
30–40	144±15	1±1	0	16±5	161±16	138 321	12.34±1.24
40–50	163±16	2±2	0	0±6	165±17	248 644	5.71±0.64
50–60	100±12	2±2	0	2±2	104±12.3	248 644	3.04±0.33
60–90	86±10	3±2	0	2±7	91±12.4	138 321	1.40±0.19
90–120	181±18	33±7	8±4	4±3	226±20	481 309	0.99±0.10
120–150	87±14	15±4	6±4	3±2	111±15.2	481 309	0.67±0.10
150–180	32±7	10±4	10±4	3±2	55±9.2	481 309	0.91±0.15

events between 120 and 160 MeV on the $E_\pi^{\rm inc}$ plot were deuterons. However, the multiple scattering is so large that this procedure is marginal for identifying any given stopping track. The average number of events in each bin was about 40 for the deuteron and 12 for the proton points. Also shown on Fig. 7 are the spreads (± one standard deviation) of the individual measured values.

The number of π-d elastic events obtained from the $E_\pi^{\rm inc}$ plots is tabulated in Table I. This number for each angular range was corrected for the following four types of events: (a) events lost via scanning inefficiency; (b) events in which the positive recoil scattered before coming to rest; (c) events where the recoil left the chamber, either by way of the glass windows or the copper wall, which made the range indeterminate.

(a) Twenty-one of the thirty rolls were double scanned. The average efficiency of (98.5±2%) for finding likely elastic events with $\theta_\pi > 40°$ was determined from the uncommon events between first and second scan. The rescan data were used along with plots of the number of events versus azimuthal angle Φ, as shown in Fig. 8, to arrive at scanning efficiency corrections. The azimuthal angle Φ is the angle between the plane of the event and the plane of the glass chamber windows ($\Phi = 90°$ would be edge-on viewing).

(b) The number of elastic events in which the deuteron scattered before coming to rest was determined by measuring events in which the recoil scattered and examining their angular correlation. A plot of $\Delta\theta_D$ of these events for each angular range was compared with the corresponding $\Delta\theta_D$ plot for events which had nonscattering recoils. This correction was quite large (approximately 20%) for $150° < \theta_\pi < 180°$; consequently, every such event was examined very closely.[15]

(c) This correction was made by measuring all events in which the recoil end point was not visible to both cameras and which could possibly have been elastic scatterings. The angular correlation of these events was used to obtain the correction except for the most backward angular range. The number of background events in the scanned region where the elastic events had in-

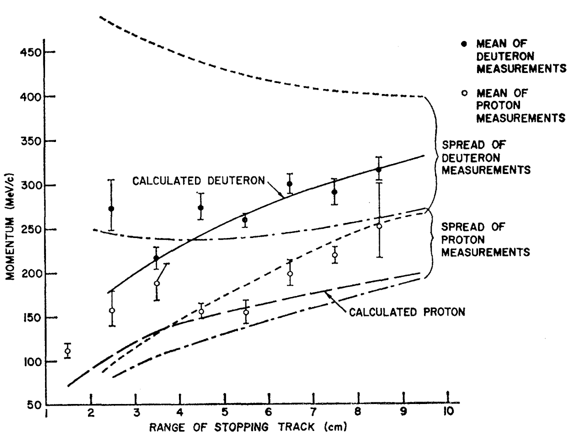

Fig. 7. Momentum of stopping protons and deuterons versus range in a field of 12.7 kG.

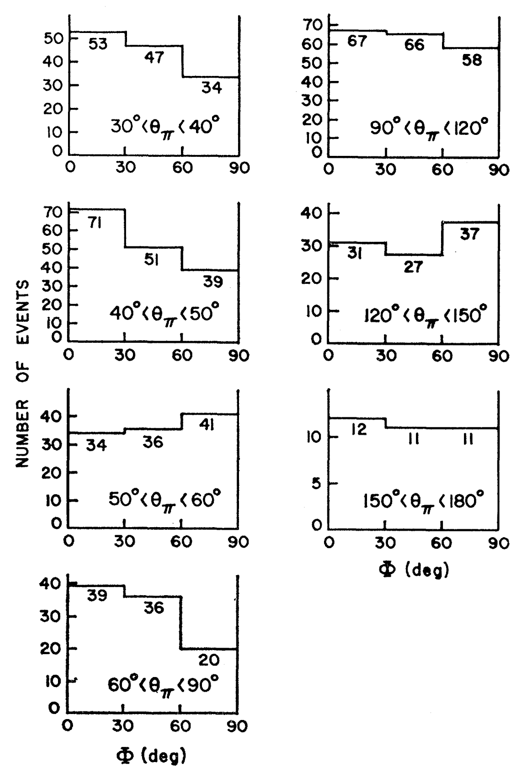

Fig. 8. Plots of azimuthal angle Φ for each angular interval.

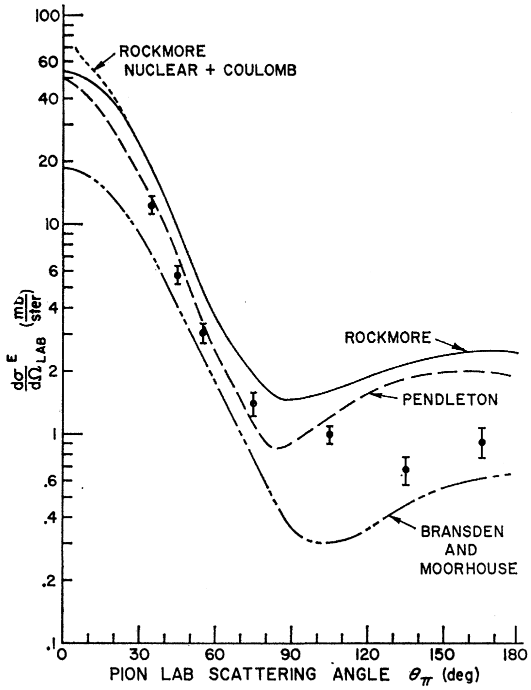

FIG. 9. Elastic differential cross sections. The experimental points are from Table I.

sufficient range to leave the chamber was used to obtain the correction for $150° < \theta_\pi < 180°$. As the windows were at $z = \pm 1.40$ in., the number of events with recoils stopping in the glass was minimized by choosing a fiducial volume bounded by $z = \pm 0.75$ in. These escape corrections were checked by a Monte Carlo computer program which effectively determined the probability of an elastic-scattering recoil leaving the chamber.

The count of the number of incident tracks in one view was corrected in order to obtain cross sections. These corrections arose from entrance window cutoff, z cutoff, beam contamination, and interaction effects. In regard to the window cutoff, $0.940\% \pm 0.004$ of the tracks which entered in one view also enter correctly on the other view. The event box had a depth cutoff of $z = \pm 0.75$ in. From a plot of all events 0.857 ± 0.021 of the events occurred inside $z = \pm 0.75$ in. The beam contamination correction was 0.89 ± 0.02.[17] The interaction correction of 0.977 ± 0.003 was obtained from the total cross section. The two remaining quantities needed to calculate a cross section are the length of each track in the chamber and the density of the deuterium which were 5.47 ± 0.05 cm and 0.131 ± 0.002 gm/cc, respectively. These quantities were used in conjunction with the number of elastic events from Table I to calculate the elastic differential cross sections which are plotted in Fig. 9.

Elastic Plus Inelastic Cross Section

Two double scanned rolls (24 000 tracks) were used to obtain the differential cross section for the sum of elastic ($\pi^- + d \rightarrow \pi^- + d$) and inelastic ($\pi^- + d \rightarrow \pi^- + p + n$) scattering. All one-prong and two-prong events found by the scanners were measured. Figure 10 is a plot of the results giving $d\sigma^{E+I}/d\Omega$ versus θ_π^{lab}. The errors are practically all statistical since the correction due to scanning inefficiency was very small. From these data, the total cross section for elastic plus inelastic with $\theta_\pi > 10°$ is 143 ± 6 mb.

Events With Uncharged Particles in Final State

The double scan of two rolls for 0-pr events enabled us to calculate the total cross section for reactions leading to neutral particles [reactions (3), (4), and (5)]. The sagitta of a 5-cm length of the incident track was measured for about half of such events. By comparing these measurements with sagitta measurements of incident beam tracks it was clearly shown that these ends were interactions in flight and not stopping tracks. The number of events with $|z| < 0.75$ in. was 171 for 31 045 uncorrected tracks. This gave a $\sigma_{\text{neutral}} = 37.2 \pm 2.8$ mb.

To obtain the charge-exchange total cross section from this result, we proceed as follows. From time-reversal invariance, charge symmetry, and $p + p \rightarrow \pi^+ + d$ cross-section data of[12] 2.5 ± 0.3 mb at 550 MeV we cal-

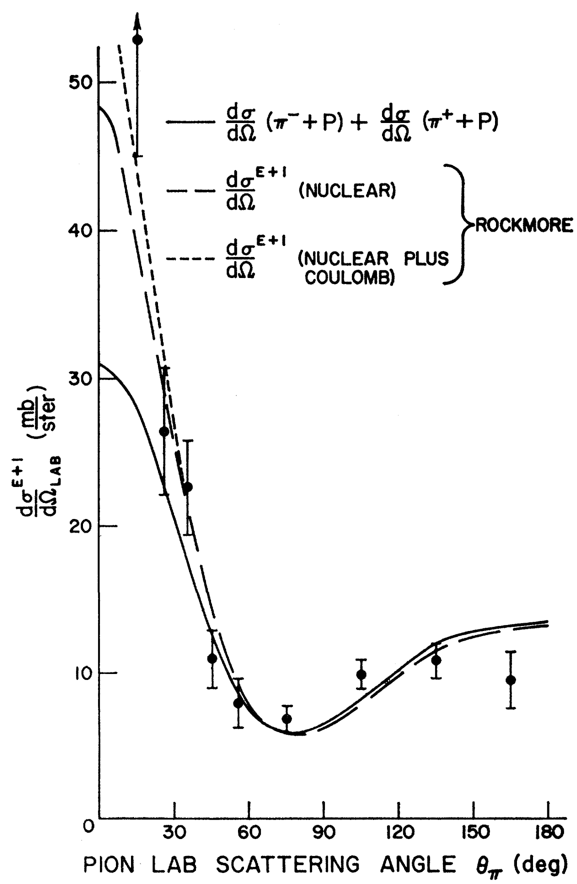

FIG. 10. Elastic plus inelastic differential cross section. The experimental points are from this experiment.

culate the cross section for $\pi^- + d \rightarrow n + n$ at 142 MeV to be 9.8 ± 1.2 mb. Radiative absorption can be estimated to be 1 ± 1 mb. Subtracting the sum of absorption and radiative absorption cross sections from σ_{neutral} yields $\sigma_{\text{charge exchange}} = 26.5 \pm 3$ mb.

The number of expected internally converted γ rays from π^0 decay for all film scanned was calculated to be 20 ± 4 using the measured internal-conversion coefficient ($2\rho_0$) of [21] 1.2% and 26.5 ± 3 mb as the charge-exchange cross section. Seventeen certain pairs and two likely pairs were found.

Total Cross Section

From Table I the cross section for elastic scattering through an angle greater than 30° is equal to 24.5 ± 1.5 mb. The cross section for elastic scattering from $\theta_\pi = 0°$ to $\theta_\pi = 30°$ was found equal to 22.5 ± 3 mb by extrapolating between the experimental estimate of the $d\sigma/d\Omega$ at $\theta_\pi = 30°$ to the $d\sigma/d\Omega$ at $\theta_\pi = 0°$ calculated by Pendleton.[14] Therefore, $\sigma_{\text{Elastic}}{}^{\text{total}} = 47 \pm 3.2$ mb. Plotted in Fig. 10 with the experimental data is the differential elastic plus inelastic cross section with and without Coulomb correction as calculated from equations of Rockmore.[5] Judging from Fig. 10, there is of the order of 1 ± 1 mb Coulomb scattering in the experimentally measured $\sigma_{E+I}(10°\text{–}180°)$. Substracting this amount and adding in $\sigma_{E+I}(0°\text{–}10°)$ obtained by extrapolating to Rockmore's zero-degree value, we obtain the nuclear cross section $\sigma_{E+I}{}^{\text{total}} = 146 \pm 6.5$ mb. Using the above elastic cross section, $\sigma_{\text{Inelastic}}{}^{\text{total}} = 99 \pm 7$ mb. The measured cross section for the reaction $\pi^- + d$ leading to only neutral outgoing particles was 37.2 ± 2.8 mb. The total interaction cross section becomes $\sigma_{\text{total}} = \sigma_{E+I} + \sigma_{\text{neutral}} = 183 \pm 7$ mb. This is to be compared with the interpolated result $\sigma_{\text{total}} = 175 \pm 8$ mb of Ashkin *et al.*,[22] and 158 ± 8 mb measured by Ignatenko *et al.*[23]

DISCUSSION

Using phase shifts,[24] the total cross section for the sum of the free-nucleon cross sections, $\sigma_t(\pi^- + p)$ plus $\sigma_t(\pi^- + n)$, is 170 mb. This is to be compared with the measured $\sigma_t(\pi^- + d) = 183 \pm 7$ mb minus 10 ± 1 mb to account for the absorption process, which has no corresponding channel for free π^--nucleon scattering.

The charge-exchange cross section for free nucleon collisions calculated from phase shifts[24] is 29.6 mb. The impulse approximation formula of Rockmore[5] gives 25.4 mb. Thus, our experimental result of 26.5 ± 3 mb is consistent with the IA calculation for the charge exchange cross section.

From Fig. 10, the total elastic plus inelastic cross section given by the IA is 145 mb. The sum of the free nucleon-scattering cross section (non-charge-exchange) equals 140 mb, again calculated from phase shifts.[24] The error on these numbers arising from uncertainties in the phase shifts is of the order of 10%. The experimental $d\sigma^{E+I}/d\Omega$ agrees with the IA calculation except for $\theta_\pi > 135°$.

Finally, in Fig. 9 is plotted the results of elastic $\pi^- + d$ calculations by Bransden and Moorhouse[6] and Pendleton,[14] along with the results obtained from equations of Rockmore. It is observed that none of the calculations agree with the experimental results over the whole angular range. The better agreement of the Bransden and Moorhouse calculation of the IA is due mainly to the unrealistic assumption that the nucleons are infinitely heavy.[8] The disagreement between the experimental points and the calculation of Pendleton, which includes a better treatment of the shortcomings of the usual IA calculation, points out the fact that there are still other corrections which may be important. As stated earlier, his calculation includes a large double-scattering correction, but no multiple scattering correction. It is likely that multiple-scattering corrections are large at this energy, on account of the large cross section for scattering in the (3, 3) state. An apparently similar disagreement between IA calculation and experiment has been pointed out by Hadjioannou[25] in the reaction $\gamma + d \rightarrow \pi^0 + d$ for the same range of momentum transfer.

With the present state of the theoretical calculations, little can be learned about the structure of the deuteron such as the D wave and repulsive-core effects from the study of π-d scattering in the (3, 3) resonance region. This experiment does provide sufficiently accurate data for comparison with more refined calculations of the π-d scattering problem.

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[22] J. Ashkin, J. P. Blaser, F. Feiner, J. G. Gorman, and M. O. Stern, Phys. Rev. **96**, 1104 (1954).

[23] A. E. Ignatenko, A. I. Mukhin, E. B. Ozerov, and B. M. Pontecorvo, Doklady Akad. Nauk. (USSR) **103**, 209 (1955).

[24] The following phase shifts were used: $\alpha_1 = 13.2°$, $\alpha_3 = -8.6°$, $\alpha_{33} = 41°$ and $\alpha_{11} = \alpha_{13} = \alpha_{31} = 0$, corresponding to an incident pion laboratory energy of 140 MeV ($\eta_B = 1.364$). α_1 and α_3 were obtained from G. Puppi [*Proceedings of the 1958 Annual International Conference on High-Energy Physics at Cern* (CERN Scientific Information Service, Geneva, 1958), p. 42]. The value of α_{33} was obtained from J. Ashkin, Suppl. Nuovo Cimento **14**, 238 (1959).

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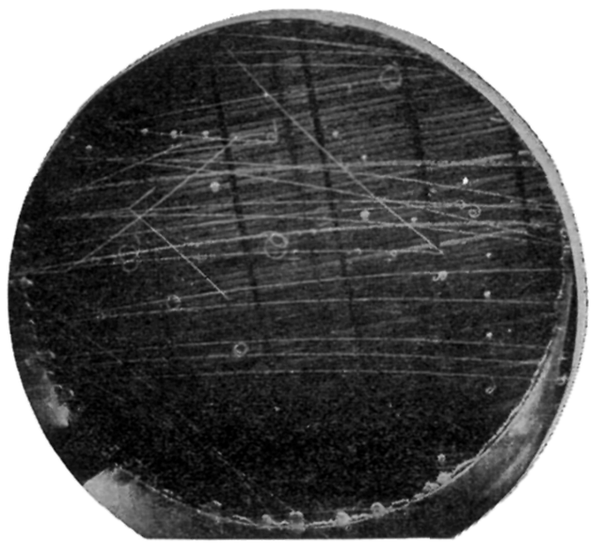

Fig. 1. A typical exposure.